

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

JEE (Advanced)

Daily Tutorial Sheet 2

11.(A) Put $t = x - [x] = \{X\}$, which is a fractional part function and lie between $0 \leq \{X\} < 1$ and then solve it.

Given, $a \in R$ and equation is $-3\{x - [x]\}^2 + 2\{x - [x]\} + a^2 = 0$

Let $t = x - [x]$, then equation is $-3t^2 + 2t + a^2 = 0 \Rightarrow t = \frac{1 \pm \sqrt{1 + 3a^2}}{3}$

$\therefore t = x - [x] = \{X\}$ [fractional part]

$\therefore 0 \leq t < 1$

$$0 \leq \frac{1 \pm \sqrt{1 + 3a^2}}{3} < 1$$

Taking positive sign, we get $0 \leq \frac{1 + \sqrt{1 + 3a^2}}{3} < 1$ [$\because \{x\} > 0$]

$$\Rightarrow \sqrt{1 + 3a^2} < 2 \Rightarrow 1 + 3a^2 < 4 \Rightarrow a^2 - 1 < 0 \Rightarrow (a + 1)(a - 1) < 0$$

$\therefore a \in (-1, 1)$, for no integer solution of a , we consider $(-1, 0) \cup (0, 1)$

12. Given, $x^2 + (a - b)x + (1 - a - b) = 0$ has real and unequal roots.

$$\Rightarrow D > 0 \Rightarrow (a - b)^2 - 4(1)(1 - a - b) > 0 \Rightarrow a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$$

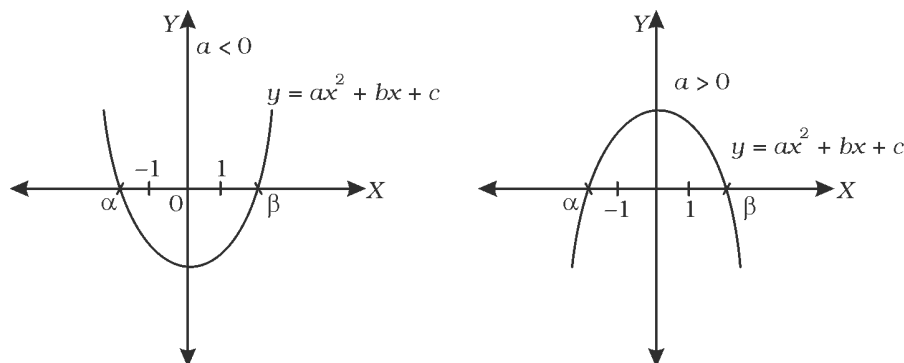
Now, to find the values of ' a ' for which equation has unequal real roots for all values of b .

i.e. Above equation is true for all b .

$\therefore$ Discriminant, $D < 0$

$$\Rightarrow (4 - 2a)^2 - 4(a^2 + 4a - 4) < 0 \Rightarrow 16 - 16a + 4a^2 - 4(a^2 + 4a - 4) < 0 \Rightarrow -32a + 32 < 0 \Rightarrow a > 1$$

13.



From figure, it is clear that, if $a > 0$, then $f(-1) < 0$ and $f(1) < 0$ and if $a < 0$, $f(-1) > 0$ and $f(1) > 0$. In both cases, $af(-1) < 0$ and $af(1) < 0$.

$$\Rightarrow a(a - b + c) < 0 \quad \text{and} \quad a(a + b + c) < 0$$

On dividing by a^2 , we get $1 - \frac{b}{a} + \frac{c}{a} < 0$ and $1 + \frac{b}{a} + \frac{c}{a} < 0$

On combining both, we get $1 \pm \frac{b}{a} + \frac{c}{a} < 0 \Rightarrow 1 + \left| \frac{b}{a} \right| + \frac{c}{a} < 0$

14.(2) (i) Given, $x^2 - 8kx + 16(k^2 - k + 1) = 0$

Now, $D = 64\{k^2 - (k^2 - k + 1)\} = 64(k - 1) > 0$

(ii) $-\frac{b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4 \Rightarrow k > 1$

(iii) $f(4) \geq 0$

$\Rightarrow 16 - 32k + 16(k^2 - k + 1) \geq 0 \Rightarrow k^2 - 3k + 2 \geq 0 \Rightarrow (k - 2)(k - 1) \geq 0$

$\Rightarrow k \leq 1$ or $k \geq 2$ Hence, $k = 2$

15.(D) Since, α is a root of $a^2x^2 + bx + c = 0 \Rightarrow a^2\alpha^2 + b\alpha + c = 0$ (i)

and β is a root of $a^2x^2 - bx - c = 0 \Rightarrow a^2\beta^2 - b\beta - c = 0$ (ii)

Let $f(x) = a^2x^2 + 2bx + 2c \therefore f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c = a^2\alpha^2 - 2a^2\alpha^2 = -a^2\alpha^2$ [from Eq. (i)]

and $f(\beta) = a^2\beta^2 + 2b\beta + 2c = a^2\beta^2 + 2b\beta^2 = 3a^2\beta^2 = 3a^2\beta^2$ [from Eq. (ii)]

$\Rightarrow f(\alpha)f(\beta) < 0$

$f(x)$ must have a root lying in the open interval (α, β) . $\therefore \alpha < \gamma < \beta$

16.(D) The equation $x^2 - px + r = 0$ has roots α, β and the equation $x^2 - qx + r = 0$ has roots $\frac{\alpha}{2}, 2\beta$.

$\Rightarrow r = \alpha\beta$ and $\alpha + \beta = p$, and $\frac{\alpha}{2} + 2\beta = q \Rightarrow \beta = \frac{2q - p}{3}$ and $\alpha = \frac{(2p - q)}{3}$

$\Rightarrow \alpha\beta = r = \frac{2}{9}(2q - p)(2p - q)$

17.(C) $(2x)^{\ell n 2} = (3y)^{\ell n 3}$

$\Rightarrow \ell n 2(\ell n 2 + \ell n x) = \ell n 3(\ell n 3 + \ell n y)$... (i)

$3^{\ell n x} = 2^{\ell n y}$

$(\ell n x)(\ell n 3) = (\ell n y)(\ell n 2)$... (ii)

$\Rightarrow \ell n 2(\ell n 2 + \ell n x) = \ell n 3\left(\ell n 3 + \frac{(\ell n x)(\ell n 3)}{\ell n 2}\right) \Rightarrow \ell n^2 2 - \ell n^2 3 = \ell n x \left\{ \frac{\ell n^2 3}{\ell n 2} - \ell n 2 \right\}$

$\Rightarrow \ell n x = -\ell n 2 \Rightarrow x = \frac{1}{2}$

18.(ABC) $x \log 3 = (x - 1) \log 4 \Rightarrow x = \frac{\log 4}{\log 4 - \log 3} = \frac{1}{1 - \log_3 4} \Rightarrow x = \frac{2 \log 2}{2 \log 2 - \log 3} = \frac{2 \log_3 2}{2 \log_3 2 - 1}$

$\Rightarrow x = \frac{2}{2 - \log_2 3}$

19.(4) Let $x = \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \sqrt{4 - \frac{1}{3\sqrt{2}} \dots}}$

$\Rightarrow x = \frac{1}{3\sqrt{2}} \sqrt{4 - x} \Rightarrow x = -\frac{1}{2}, \frac{4}{9}$

$x = \frac{4}{9}$

$$\therefore 6 + \log_{3/2}(4/9) = 4$$

20.(D) $f(x) = x^{12} - x^9 + x^4 - x + 1$

When $x \leq 0 \Rightarrow x^{12} > 0, -x^9 > 0, x^4 > 0, -x > 0 \quad \therefore f(x) > 0 \quad \forall x \leq 0$

When $0 < x \leq 1 \Rightarrow x^{12} > 0, x^4 - x^9 > 0, 1 - x > 0 \quad \therefore f(x) > 0 \quad \forall x \in (0, 1]$

When $x > 1 \Rightarrow x^{12} - x^9 > 0, x^4 - x > 0 \quad \therefore f(x) > 0$